The Physics of Quantum Computing

Postulates of Quantum Mechanics

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Conventional Computers Properties And Characteristics

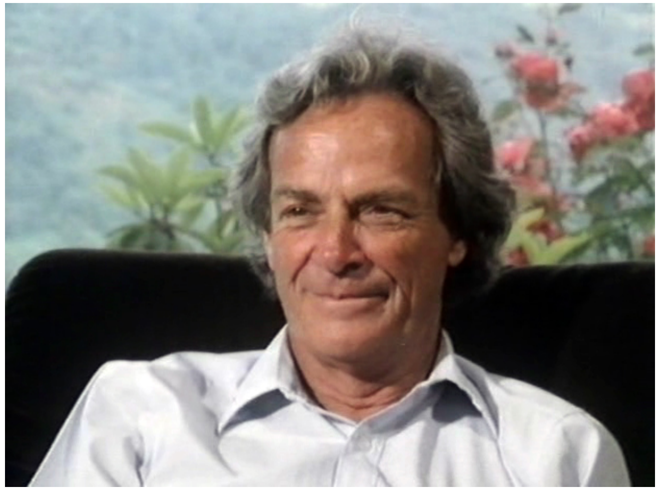
Basic Characteristic of a Classical Computer

- Binary data representation for floating point and integer quantities ("0"s and "1"s)

- Hardware is designed and constructed on this base 2 formalism

- Binary representations reflect the lowest level structure for system and application software

Constraint of the Digital Computing Approach



Richard Feynman (1981):

"...trying to find a computer simulation of physics, seems to me to be an excellent program to follow out...and I'm not happy with all the analyses that goes with just the classical theory, because

- *nature isn't classical*, dammit
- if you want to make a simulation of nature, you'd better *make it quantum mechanical*, and by golly it's a wonderful problem because it doesn't look so easy."

Richard Feynman's 1981 Paper

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

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1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there's no implication that anybody needs to talk about the same thing or anything like it. So what I want to talk about is what Mike Dertouzos suggested that nobody would talk about. I want to talk about the problem of simulating physics with

The Quantum Computer
A New Computational Paradigm

David Deutsch (1985):

"Computing machines resembling the universal quantum computer could, in principle, be built and would have many remarkable properties not reproducible by any Turing machine … Complexity theory for [such machines] deserves further investigation."

Challenges Using the Physics of Quantum Mechanics to Construct a Quantum Computer

Quantum Mechanics and Computing

If one wants to use quantum mechanics to build a computer, one must understand and appreciate the implications how a quantum computer will view and process the problem

Challenges Conceptualizing How a Quantum Computer Operates

- Quantum mechanics is not a description of the classical world
- It describes the physics of the atomic and subatomic world
- Difficult conceptually
  - Our human ideas and approaches to problems are influenced by our experiences and expected behaviors
  - All known human experiences and intuition is rooted in our classical world
- Many behaviors in the quantum world have no classical analog

Quantum Computing Challenges

Even if an algorithm or program can be shown to be based on the postulates of quantum mechanics it must also be demonstrated that the quantum mechanical algorithm is computationally superior to the classical equivalent

Quantum Supremacy

Quantum supremacy is the potential ability of quantum computing devices to solve problems that classical computers practically cannot (measured as superpolynomial speedup over the best known classical algorithm)

Postulates of Quantum Mechanics

Postulate 1

1. The totality of the mathematical representation of the state of a system can be quantum mechanically represented by a **ket** $|\boldsymbol{\Psi}>$ in the space of states

Postulate 1 Implications for Quantum Computing

Mathematical representation of a quantum system

- Every isolated system has an associated complex vector space with an inner product that is the state space of the system

- A unit vector in the system's state space is a state vector that is a complete description of the physical system

Dirac “bra” and “ket” Notation

- Many texts use Dirac “ket” notation |a> to represent a column vector

$$|a\rangle = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

and a Dirac “bra” notation to denote the Hermitian conjugate of $\vec{\boldsymbol{a}}$

$$< a| = (a_1^* \quad a_2^* \quad \ldots \quad a_n^*)$$

The **transpose $\mathbf{a}^T$** of a column vector a is a row vector
The **adjoint $\boldsymbol{a}^\dagger$** is the complex conjugate transpose of a column vector a and is sometimes called the Hermitian conjugate
Unitary matrix U is a complex square matrix whose adjoint equals its inverse and the product of U adjoint and the matrix U is the identity matrix

$$U^\dagger U = U^{-1} U = I$$

Postulate 1 Implications for Quantum Computing

- This postulate implies that the superposition of two states in the Hilbert Space A is again a state of the system.

- Composite System

Given that the Hilbert space of system A is H_A and the Hilbert space of system B is H_B , then the Hilbert space of the composite systems AB is the "tensor product" $H_A \otimes H_B$

Tensor Product from Matrices

- Let A and B be represented by the following matrices

$$A = \begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix} \qquad B = \begin{pmatrix} \boldsymbol{e} & \boldsymbol{f} \\ \boldsymbol{g} & \boldsymbol{h} \end{pmatrix}$$

$$A \otimes B = \begin{bmatrix} a \begin{pmatrix} \boldsymbol{e} & \boldsymbol{f} \\ \boldsymbol{g} & \boldsymbol{h} \end{pmatrix} & b \begin{pmatrix} \boldsymbol{e} & \boldsymbol{f} \\ \boldsymbol{g} & \boldsymbol{h} \end{pmatrix} \\ c \begin{pmatrix} \boldsymbol{e} & \boldsymbol{f} \\ \boldsymbol{g} & \boldsymbol{h} \end{pmatrix} & d \begin{pmatrix} \boldsymbol{e} & \boldsymbol{f} \\ \boldsymbol{g} & \boldsymbol{h} \end{pmatrix} \end{bmatrix}$$

Another Surprising Example of Quantum Behavior Quantum Entanglement

- Quantum entanglement is a phenomenon in quantum mechanics when
  - pairs (groups) of particles are generated and/or interact such that
  - Their quantum mechanical individual states cannot be mathematically described independently of the pair (group) state

Entanglement
Mathematical Framework

- Given two non-interacting systems A and B described by Hilbert spaces $\mathbb{H}_\mathrm{A}$ and $\mathbb{H}_\mathrm{B}$ the composite system is expressed as

$$\mathbb{H}_\mathrm{A} \otimes \mathbb{H}_\mathrm{B}$$

- The state of the composite system is

$$| \Psi_\mathrm{A} > \otimes \, | \Psi_\mathrm{B} >$$

- States of $\mathbb{H}_\mathrm{A}$ and $\mathbb{H}_\mathrm{B}$ that can be mathematically represented in this manner are called separable states or product states

$$|\Psi >_{AB} = \sum_{i,j} c_{ij} \, |i >_A \otimes \, |j >_B$$

Quantum Entanglement Basis States

- Define a basis vectors $|i>_A$ for $\mathbb{H}_A$ and $|j>_B$ for $\mathbb{H}_B$

- The composite (product state) can be written in the set of basis vectors as

$$|\Psi>_{AB} = \sum_{i,j} c_{ij} \, |i>_A \otimes |j>_B$$

$$|\Psi>_A = \sum_i c_i^A \, |i>_A \qquad |\Psi>_B = \sum_j c_j^B \, |j>_B$$

- If there exist vectors c_i^A , c_j^B such that $c_{ij} = c_i^A \, c_j^B$ for all states then the system is considered separable

Quantum Entanglement
Basis States

- If there is at least one pair c_i^A , c_j^B such that $c_{ij} \neq c_i^A\, c_j^B$ then the state is labelled as being entangled

- Example $\frac{1}{\sqrt{2}}\,(|0>_A \otimes |1>_B - (|1>_A \otimes |0>_B\,)$

Possible Outcomes for an Entangled System

$$\frac{1}{\sqrt{2}}(|0>_A \otimes |1>_B - (|1>_A \otimes |0>_B))$$

- 2 observers (Alice and Bob) and a 2 state basis set {|0>, |1>}
- Alice is an observer in system A and Bob is an observer in system B
- Alice makes an observation in {|0>, |1>} basis → 2 equal outcomes
  - If Alice measures |0> , then system states collapses to $|0>_A|1>_B$ and Bob must measure the |1> state
  - If Alice measures |1> then system states collapses to $|1>_A|0>_B$ and Bob must measure the |0> state

- This will happen regardless of the spatial separation of system A and B
- Completely unexpected behavior compared to everyday human experiences of causality and locality

Postulate 2

2. Every observable attribute of a physical system is described by an operator that acts on the kets that describe the system.

Postulate 2 Implications for Quantum Computing

- Acting with an operator on a state in general changes the state.

- There are special states that are not changed (except for being multiplied by a constant) by the action of an operator

$$\mathbb{A}|\Psi_a> = a|\Psi_a>$$

- The numbers "a" are the eigenvalues of the eigenstates

Postulate 3

3. The only possible result of the measurement of an observable "O" is one of the eigenvalues of the corresponding operator "$\hat{O}$".

Postulate 3 Implications for Quantum Computing

- This postulate is the basis for describing the discreteness of measured quantities i.e. “quantized”
- Experimental measurements are described by real numbers

  ➔ the eigenvalues of quantum operators describing the real world must be Hermitian
- Hermitian operators are orthogonal ➔ $<a_j|a_k> = \delta_{jk}$
- They span the space ➔ they form a basis
  - An arbitrary state can be expanded as a sum of the eigenstates of a Hermitian operator (with complex coefficients)
  - This implies the property that the set of states are “complete”

Postulate 4

- When a measurement of an observable $\mathbb{A}$ is made on a generic state $|\Psi>$, the probability of obtaining an eigenvalue a_{n} is given by the square of the inner product of $|\Psi>$ with the eigenstate $|a_{\mathrm{n}}>$, $|<a_{\mathrm{n}}|\psi>|^2$

Postulate 4 - Implications for Quantum Computing

- The complex number $<a_n|\Psi>$ is a “probability amplitude”
  Note: This quantity is not directly measureable
- To obtain an expectation value must square the probability amplitude
- The probability of obtaining some result must be 1.

$$|<\Psi|\Psi>|^2 = \sum_m \sum_n c^*_m c_n < a_m | a_n >$$

- There are complex coefficients in the probability amplitude that must be summed and then multiplied to obtain the expectation value

Postulate 5

5. The operator $\mathbb{A}$ corresponding to an observable that yields a measured value "a_n" will correspond to the state of the system as the normalized eigenstate $|a_n>$

Postulate 5 Implications for Quantum Computing

- This postulate describes the collapse of the wave packet of probability amplitudes when making a measurement on the system
- A system described by a wave packet $|\Psi>$ and measured by an operator $\mathbb{A}$ repeated times will yield a variety of results given by the probabilities $|<a_n|\Psi>|^2$
- If many identically prepared systems are measured each described by the state |a> then the expectation value of the outcomes is

$$< a >\equiv \sum_n a_n \, \mathrm{Pr}ob(a_n) =< a|\mathbb{A}|a>$$

Digital Computer Measurements Versus Quantum Computing Measurements

- Quantum mechanics probability amplitude is a complex valued unobservable described by a state vector (wavefunction)

- The probability amplitude has an indeterminate specific value until a measurement is performed

- A measurement collapses the wave packet of all possible probability amplitudes down to a single measurement while preserveing the normalization of the state

- Once the system is measured all information prior to that measurement is permanently lost

Digital Computer Measurements Versus Quantum Computing Measurements

- Any direct disruptions of the of the quantum computing calculation will immediately select/collapse the system to a single value state – all information prior to the measurement is lost

- Digital computing practices of inserting
  - Intermediate print statements
  - Checkpoint re-starts

  disallowed by quantum mechanics in a quantum computer

Digital Computer Measurements Versus Quantum Computing Measurements

- Quantum computers output probabilities (expectation values)
- Quantum computer output probability distribution of results for the calculation given by $|\langle a_n| \Psi\rangle|^2$

- Quantum computer outputs are statistically independent
- Cannot re-run the quantum computing program a 2^{nd} time and always expect to get exactly same answer

Postulate 6

Dynamics - Time Evolution of a Quantum Mechanical System

- The evolution of a closed system that evolves over time is expressed mathematically by a unitary operator that connects the system between time t_1 to time t_2 and that only depends on the times t_1 and t_2

- The time evolution of the state of a closed quantum system is described by the Schrodinger equation

$$i\hbar \frac{d}{dt}|\Psi >= H(\mathrm{t})|\Psi >$$

Postulate 6 Implications for Quantum Computing

- Any type of “program” that would represent a step by step evolution from an initial state on a quantum computer to some final state must preserve the norm of the state (conservation of probability)

- Requirement that each “step-by-step” evolution must preserve unitarity (forces constraints for “programming” a quantum computer)

- The requirement of postulate 6 that the quantum mechanical system be closed for this unitary evolution of the system over time (forces constraints for “programming” a quantum computer)

Questions

Representing Information on a Computer

- Computer has two states ("off" and "on")
- Define two states "0" and "1" ("bits")
- Need to be able to represent the state of a system on a computer in only terms of "0"s and "1"s
- Need to understand how these "0"s and "1"s can be manipulated – how they are transformed when an operation is applied to them

Single Component Representation

- Identify general rules for transforming the state of a single bit in every possible way.
- NOT gate

Initial State		Final State
0	not(0)	1
1	not(1)	0

- RESET gate - Sets the state to 0 regardless of the input

Initial State		Final State
0	reset(0)	0
1	reset(1)	0

- These two operations define all possible ways to transform the state of a single bit